Chapter 2.3: Polynomial Functions and Their Graphs

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

constant

a_n is the leading coefficient and the polynomial has a degree of n.

Polynomial Functions are both smooth and continuous

End Behaviors:

Even Degree:

Leading Coefficient >0

Right Rises and Left Rises

$(\infty,\infty)\cup(-\infty,\infty)$

Leading Coefficient <0

Right Falls and Left Falls

$(\infty,-\infty)\cup(-\infty,-\infty)$

Odd Degree:

Leading Coefficient >0

Right Rises and Left Falls

$(\infty,\infty)\cap(-\infty,-\infty)$

Leading Coefficient <0

Right Falls and Left Falls

$(\infty,-\infty)\cap(-\infty,-\infty)$

Use the leading coefficient test to determine the end behavior of the polynomial.

$$f(x) = x^3 + 3x^2 - x - 3$$

$$f(x) = -143x^3 + 1810x^2 - 187x + 2331$$

$$f(x) = -x^4 + 8x^3 + 4x^2 + 2$$

Find the zeros: $f(x) = x^3 + 3x^2 - x - 3$

zeros
roots
x-int
sol.

$$\underbrace{x^3 + 3x^2}\ \underbrace{-x - 3} = 0$$

$$x^2(\underline{x+3}) - 1(\underline{x+3}) = 0$$

$$(x+3)(x^2-1) = 0$$

$$(x+3)(x+1)(x-1) = 0$$

$$\boxed{x = -3, -1, 1}$$

Find the zeros: $f(x) = -x^4 + 4x^3 - 4x^2$

$-x^4 + 4x^3 - 4x^2 = 0$

$-x^2(x^2 - 4x + 4) = 0$

$-x^2(x-2)(x-2) = 0$

$-x^2 = 0$ $\quad x - 2 = 0$ $\quad x - 2 = 0$

$x = 0$ $\quad x = 2$ $\quad x = 2$

$\boxed{x = 0, 2}$ M2 M2

Multiplicity and X-Intercepts:

If r is a zero of even multiplicity, the graph touches the x-axis and turns around.

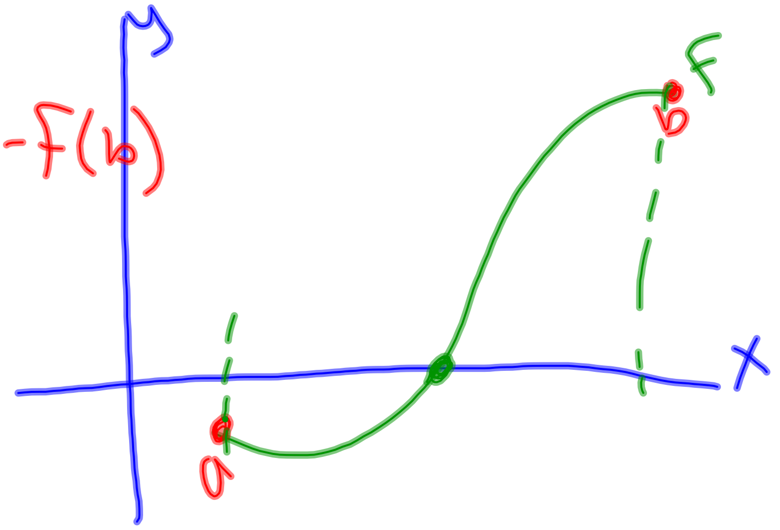
If r is a zero of ~~even~~ odd multiplicity, the graph passes through the x-axis.

The bigger the multiplicity the flatter the graph is at the zero...

Intermediate Value Theorem:

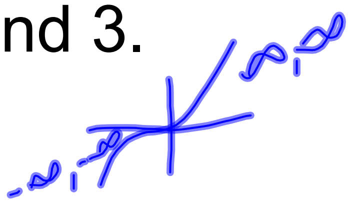
If f is a polynomial and f(a) and f(b) have opposite signs then there is a real zero between a and b.

$f(a)$ $-f(b)$



Show that the polynomial has a real zero between 2 and 3.

$$f(x) = x^3 - 2x - 5$$



$$f(2) = 2^3 - 2(2) - 5 = -1$$

$$f(3) = 3^3 - 2(3) - 5 = 16$$

Turning Points:

a polynomial of degree n has at most n-1 turning points.

x^3 can have less....

Symmetry:

$f(-x) = f(x)$ $f(-x) = -f(x)$

Even: over y-axis Odd: Over Origin

graph: $f(x) = x^4 - 2x^2 + 1$

EB: $(\infty, \infty) \cup (-\infty, \infty)$

X-Int: $x^4 - 2x^2 + 1 = 0$ $(x^2 - 1)(x^2 - 1) = 0$

$(x+1)(x-1)(x+1)(x-1) = 0$

$x = 1, -1$ M2 M2

Y-Int: $(0, 1)$

Symmetry: $(-x)^4 - 2(-x)^2 + 1$ over y-axis

$x^4 - 2x^2 + 1$

Turning Points: 3 @ $(-1,0)(0,1)(1,0)$

Suggested: pg. 278
#'s 7,18,23,31,39,41,45,57